FORMULAS FOR REFERENCE

SPHERE	Surface area	=	$4\pi r^2$
	Volume	=	$\frac{4}{3}\pi r^3$
CYLINDER	Area of curved surface	=	$2\pi rh$
	Volume	=	$\pi r^2 h$
CONE	Area of curved surface	=	πrl
	Volume	=	$\frac{1}{3}\pi r^2 h$
PRISM	Volume	=	base area $\times$ height
PYRAMID	Volume	=	$\frac{1}{3}\times$ base area $\times$ height

SECTION A **Answer ALL questions in this section.**
There is no need to start each question in this section on a fresh page.
Geometry theorems need not be quoted when used.

1. If $3x^2 - kx - 2$ is divisible by $x - k$, where k is a constant, find the two values of k.

(5 marks)

2. The table below shows the distribution of the marks of a group of students in a short test :

Marks	1	2	3	4	5
Number of Students	10	10	5	20	x

If the mean of the distribution is 3, find the value of x.

(5 marks)

3. Expand $(1+\sqrt{2})^4$ and express your answer in the form $a + b\sqrt{2}$ where a and b are integers.

(5 marks)

4. Factorize

(a) $x^2y + 2xy + y$,

(b) $x^2y + 2xy + y - y^3$.

(6 marks)

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5.

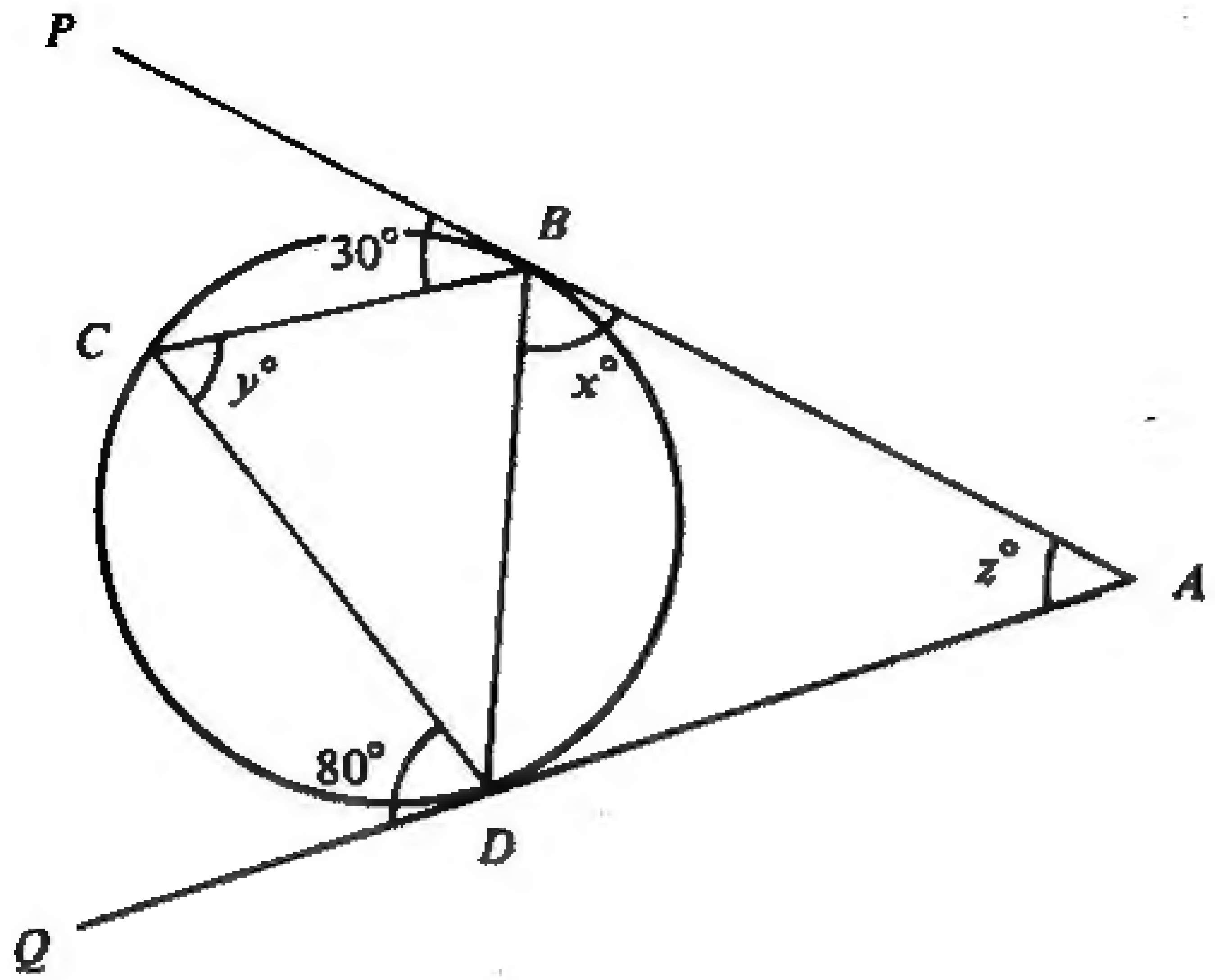

Figure 1

In Figure 1, AP and AQ touch the circle BCD at B and D respectively. $\angle PBC = 30^\circ$ and $\angle CDQ = 80^\circ$. Find the values of x, y and z.

(6 marks)

6. Solve $x - 5\sqrt{x} - 6 = 0$.

(6 marks)

7. Given $\tan\theta = \dfrac{1+\cos\theta}{\sin\theta}$ $(0^\circ < \theta < 90^\circ)$,

(a) rewrite the above equation in the form $a\cos^2\theta + b\cos\theta + c = 0$ where a, b and c are integers;

(b) hence, solve the given equation, giving your answer in degrees.

(6 marks)

SECTION B Answer FIVE questions in this section. Each question carries 12 marks.

8.

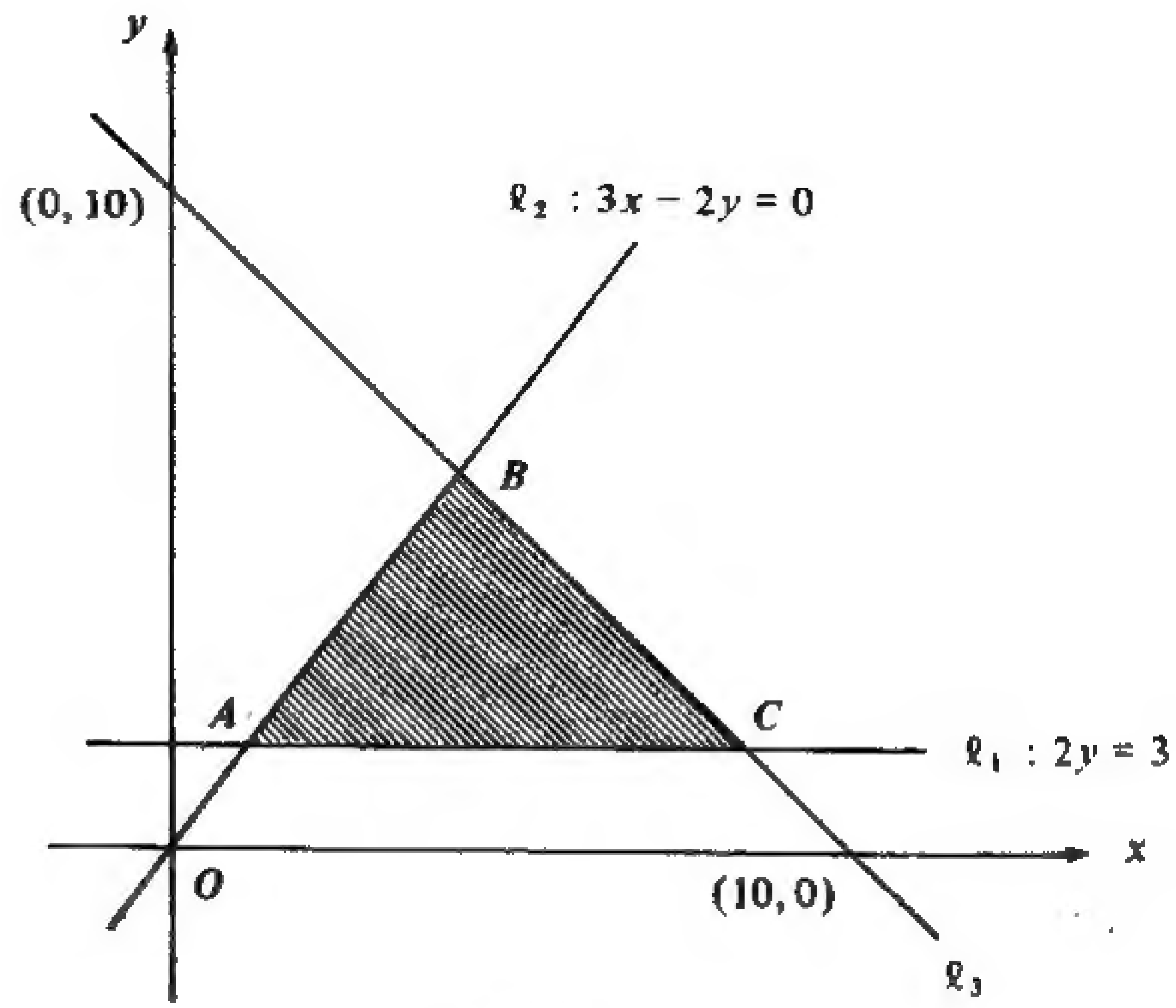

Figure 2

In Figure 2, $\ell_1 : 2y = 3$,

$\ell_2 : 3x - 2y = 0$.

The line ℓ_3 passes through $(0, 10)$ and $(10, 0)$.

(a) Find the equation of ℓ_3.

(2 marks)

(b) Find the coordinates of the points A, B and C.

(3 marks)

(c) In Figure 2, the shaded region, including the boundary, is determined by three inequalities. Write down these inequalities.

(3 marks)

(d) (x, y) is any point in the shaded region, including the boundary, and $P = x + 2y - 5$. Find the maximum and minimum values of P.

(4 marks)

Go on to the next page

9. Let L be the line $y = k - x$ (k being a constant) and C be the circle $x^2 + y^2 = 4$.

(a) If L meets C at exactly one point, find the two values of k.

(6 marks)

(b) If L intersects C at the points $A(2, 0)$ and B,

(i) find the value of k and the coordinates of B;

(ii) find the equation of the circle with AB as diameter.

(6 marks)

10. a and b are positive numbers. $a, -2, b$ form a geometric progression and $-2, b, a$ form an arithmetic progression.

(a) Find the value of ab.

(2 marks)

(b) Find the values of a and b.

(5 marks)

(c) (i) Find the sum to infinity of the geometric progression $a, -2, b, \ldots$.

(ii) Find the sum to infinity of all the terms that are positive in the geometric progression $a, -2, b, \ldots$.

(5 marks)

11. (a) There are two bags. Each bag contains 1 red, 1 black and 1 white ball. One ball is drawn randomly from each bag. Find the probability that

(i) the two balls drawn are both red;

(ii) the two balls drawn are of the same colour;

(iii) the two balls drawn are of different colours.

(6 marks)

(b) A box contains 2 red, 2 black and 3 white balls. One ball is drawn randomly from the box. After putting the ball back into the box, one ball is again drawn randomly. Find the probability that

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(6 marks)

12.

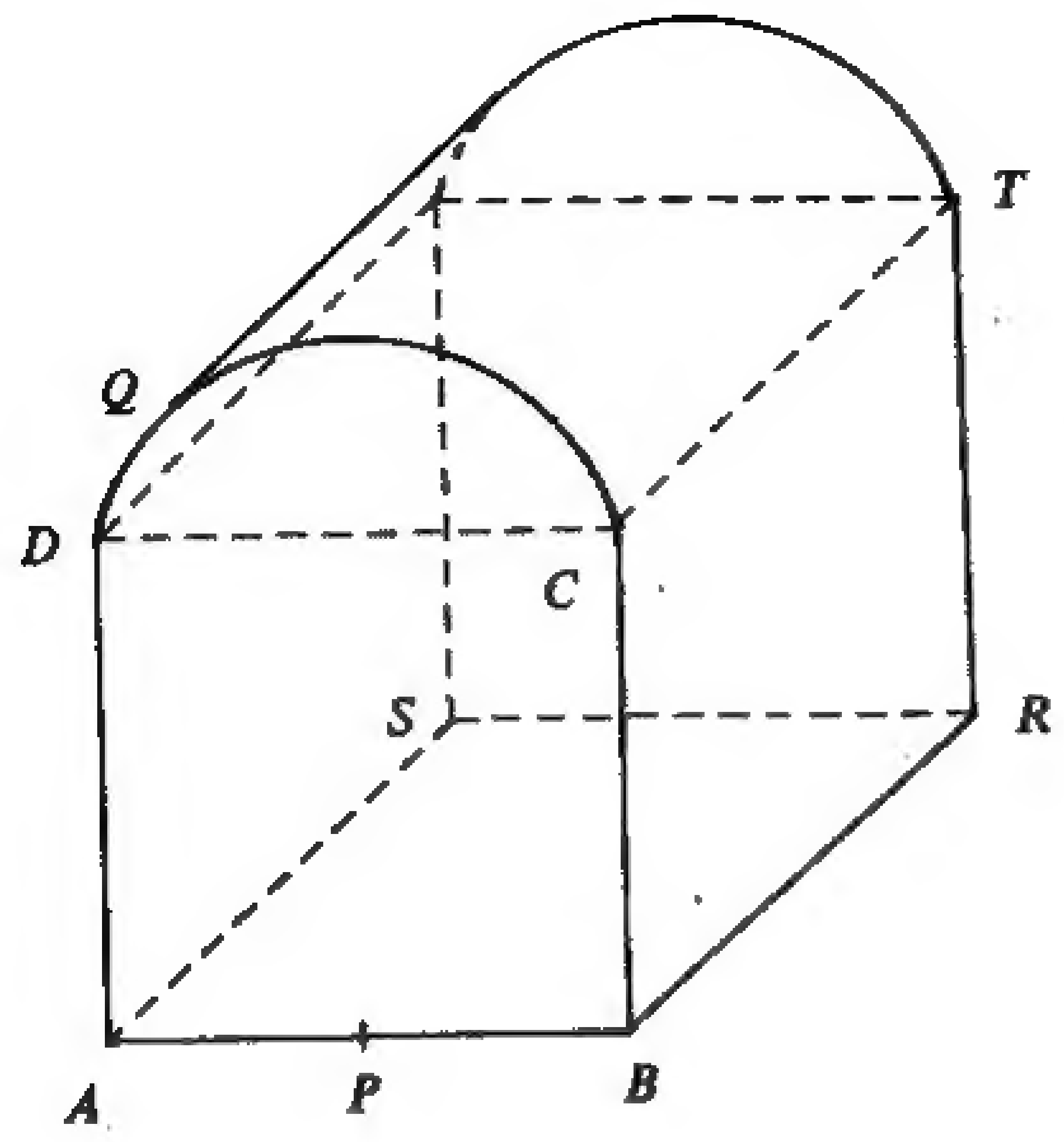

Figure 3

In Figure 3, all vertical cross-sections of the solid that are parallel to $APBCQD$ are identical. $ABCD$, $BRTC$ and $ABRS$ are squares, each of side 20 cm. P is the mid-point of AB. CQD is a circular arc with centre P and radius PC.

(In this question, give your answers correct to 1 decimal place.)

(a) Find, in degrees, $\angle CPD$. (3 marks)

(b) Find, in cm, the length of the arc CQD. (3 marks)

(c) Find, in cm^2, the area of the cross-section $APBCQD$. (3 marks)

(d) Find, in cm^2, the total surface area of the solid. (3 marks)

13.

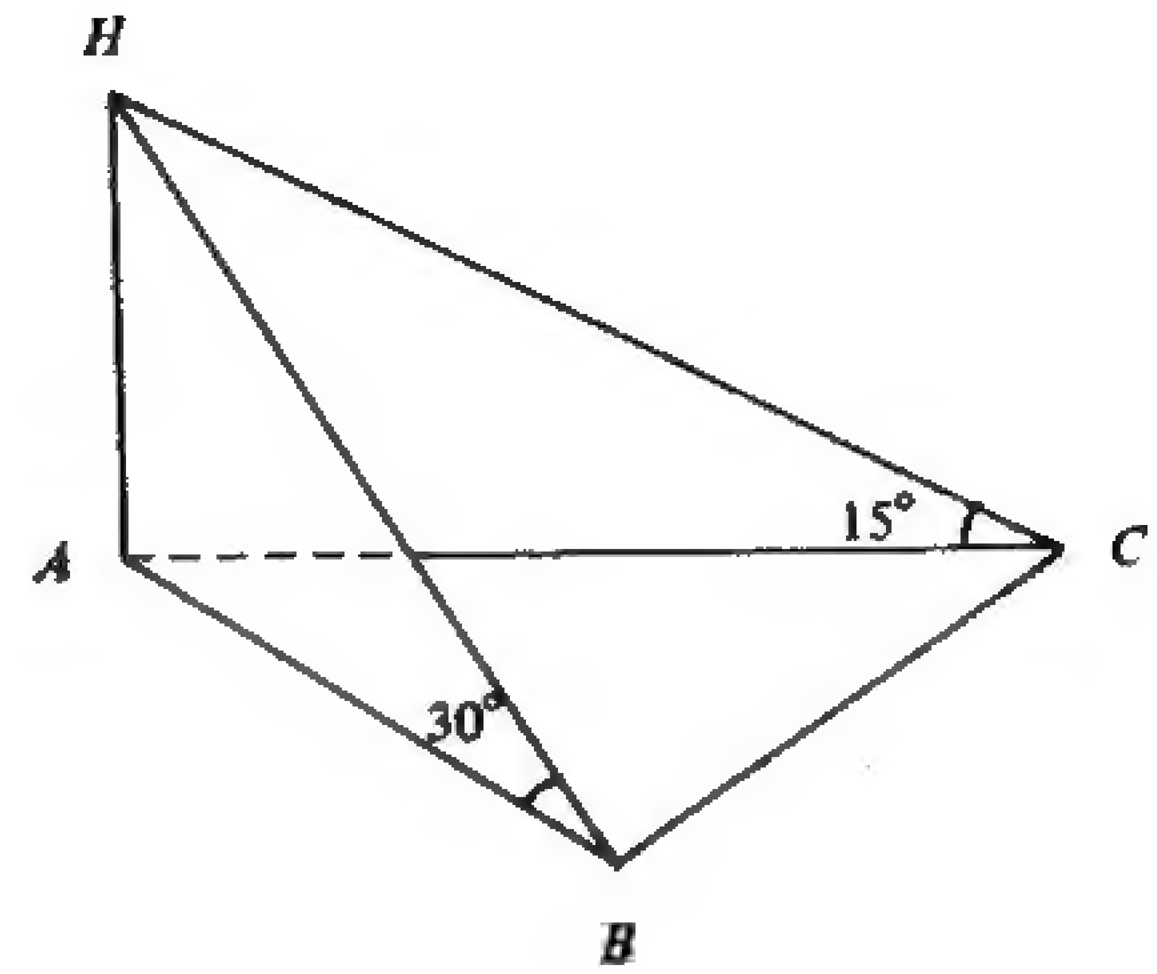

Figure 4

In Figure 4, A, B and C lie in a horizontal plane. $AC = 20$ m. HA is a vertical pole. The angles of elevation of H from B and C are 30° and 15° respectively.

(In this question, give your answers correct to 2 decimal places.)

(a) (i) Find, in m, the length of the pole HA.

(ii) Find, in m, the length of AB. (6 marks)

(b) If A, B and C lie on a circle with AC as diameter,

(i) find, in m, the distance between B and C;

(ii) find, in m^2, the area of ΔABC. (6 marks)

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14. (a) Figure 5 shows the graph of $y = x^3 + x^2$ for $-1 \leqslant x \leqslant 2$.

(i) Draw a suitable straight line in Figure 5 and use it to find a root of the equation

$$x^3 + x^2 + x - 4 = 0 .$$

(Give your answer correct to 1 decimal place.)

(ii) By the method of magnification, find the root obtained in (i) correct to 2 decimal places.

(7 marks)

(b) A bank introduces the following savings scheme in which interest is compounded yearly:

If a customer deposits \$2500 on the first day of each year for three successive years, he will receive \$10 000 at the end of the third year.

Assume that the interest rate is $r\%$ per annum.

(i) Show that

$$(1 + r\%)^3 + (1 + r\%)^2 + (1 + r\%) = 4 .$$

(ii) Find, correct to 2 significant figures, the value of r, by using the results in (a) (ii) and (b) (i).

(5 marks)

Candidate No.	Centre No.	Seat No.	Total Marks on this page	

14. **If you attempt Question 14, fill in the details in the first three boxes above and tie this sheet into your answer book.**

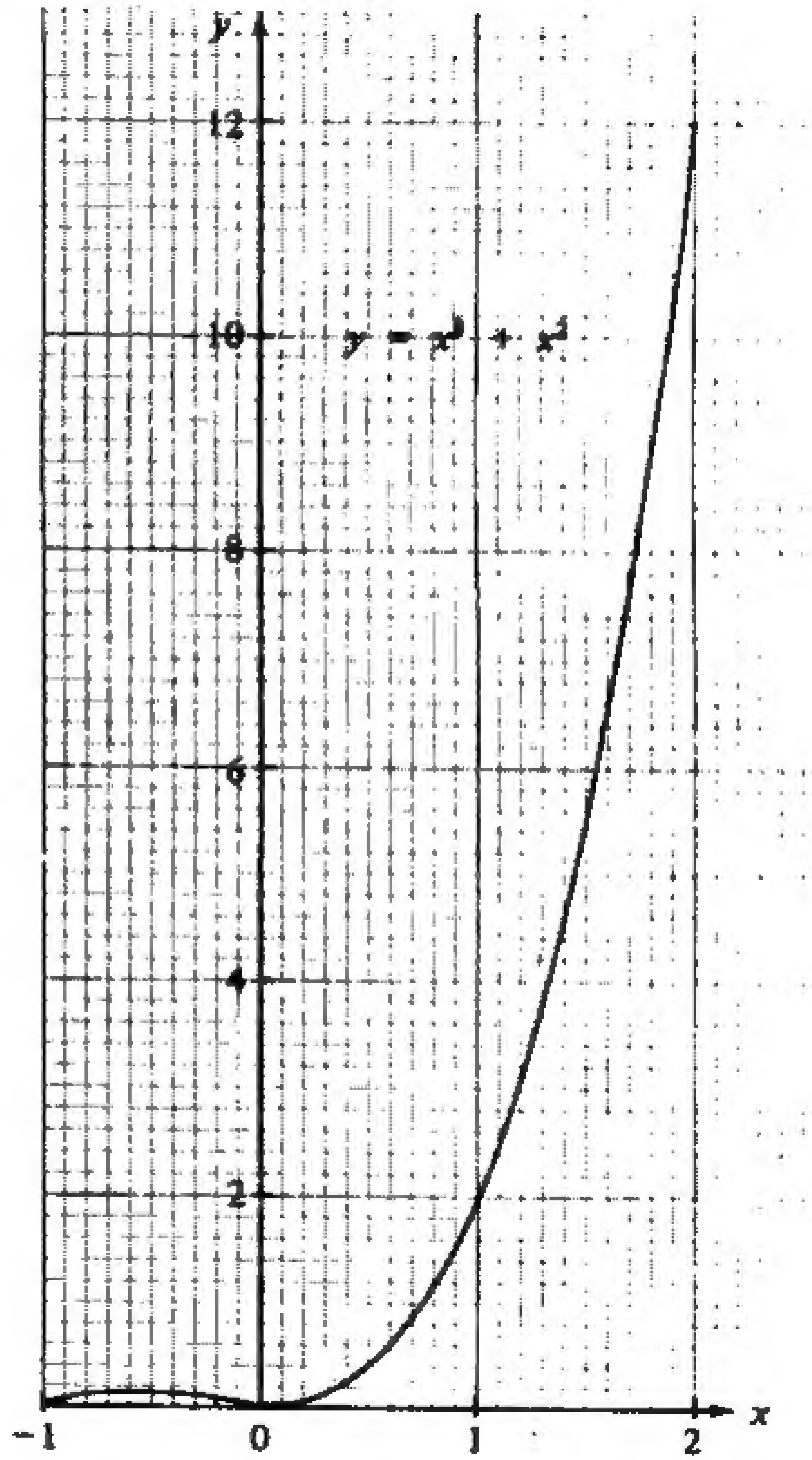

Figure 5

END OF PAPER

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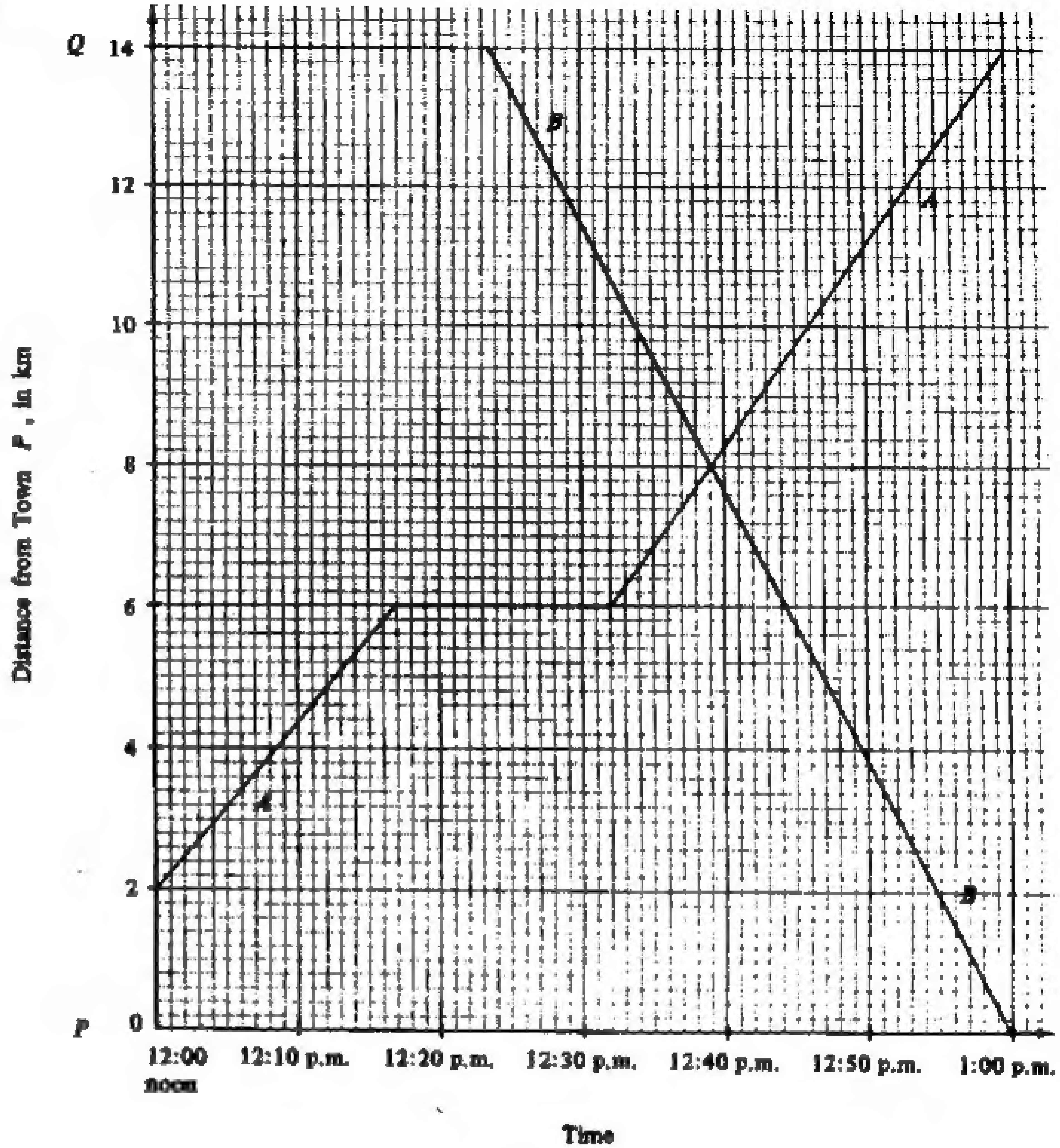

Figure 1

Figure 1 shows the travel graphs of two cyclists A and B travelling on the same road between towns P and Q, 14 km apart.

(a) For how many minutes does A rest during the journey ?

(b) How many km away from P do A and B meet ?

(5 marks)

Go on to the next page

4. Factorize

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(6 marks)

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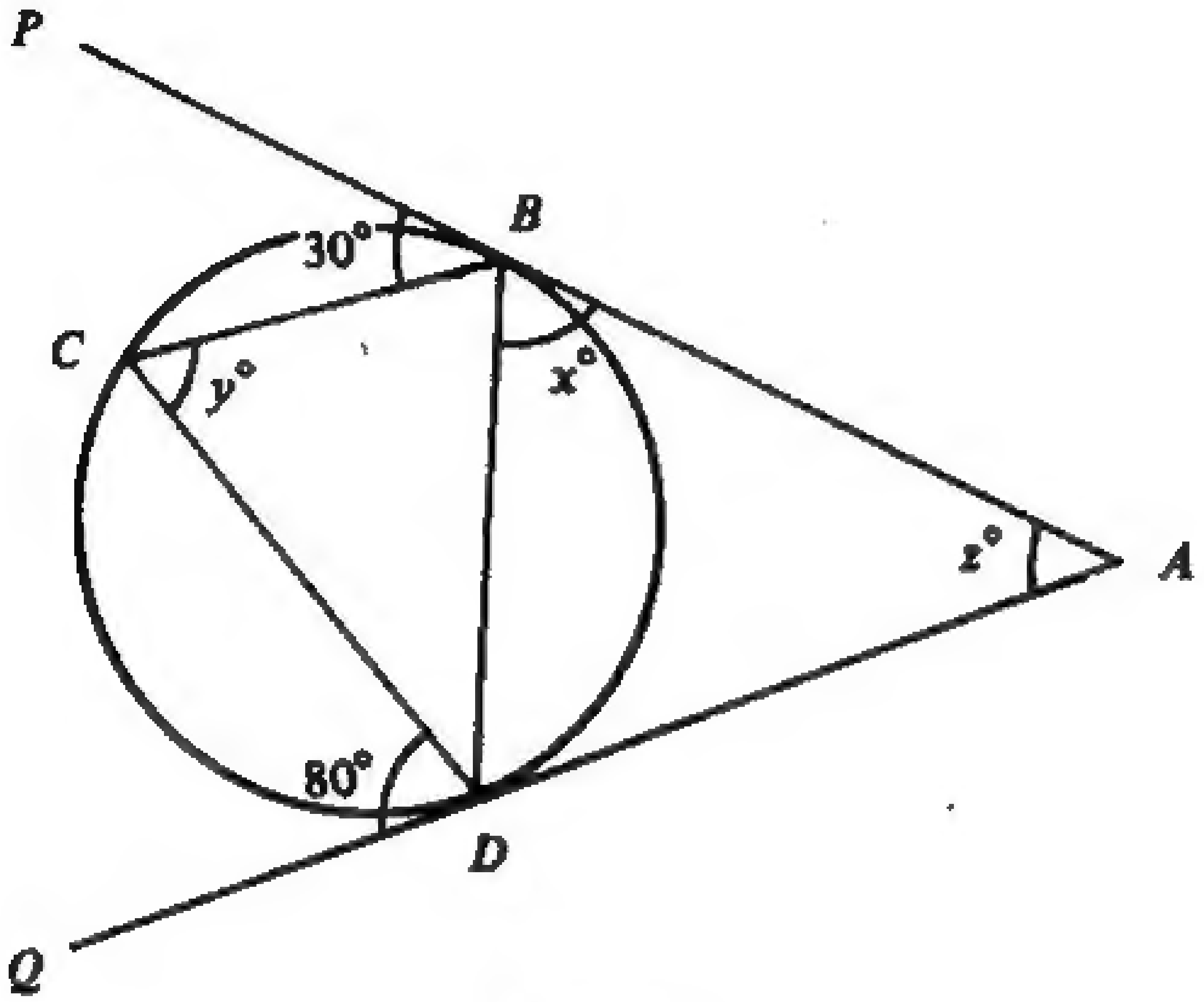

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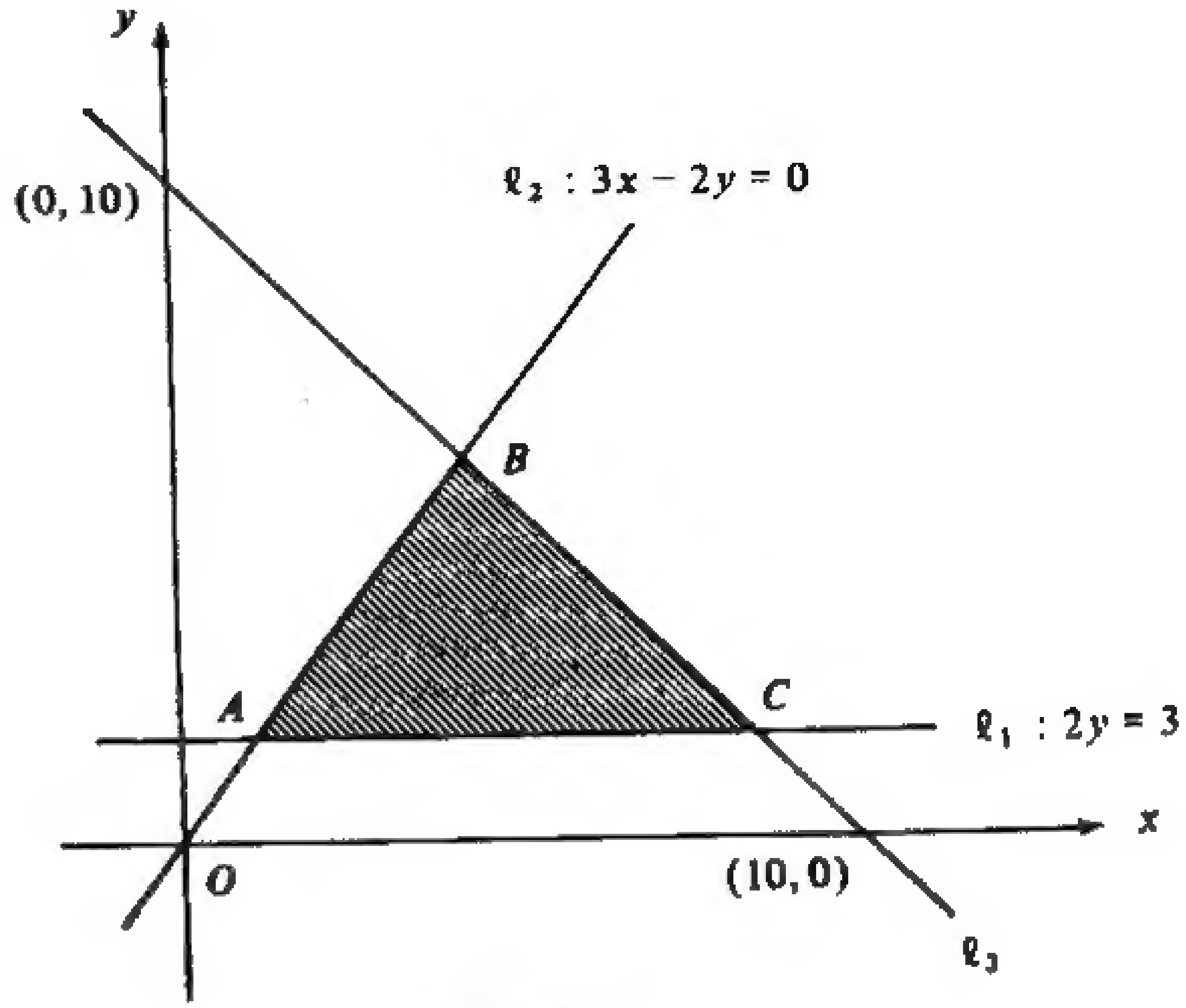

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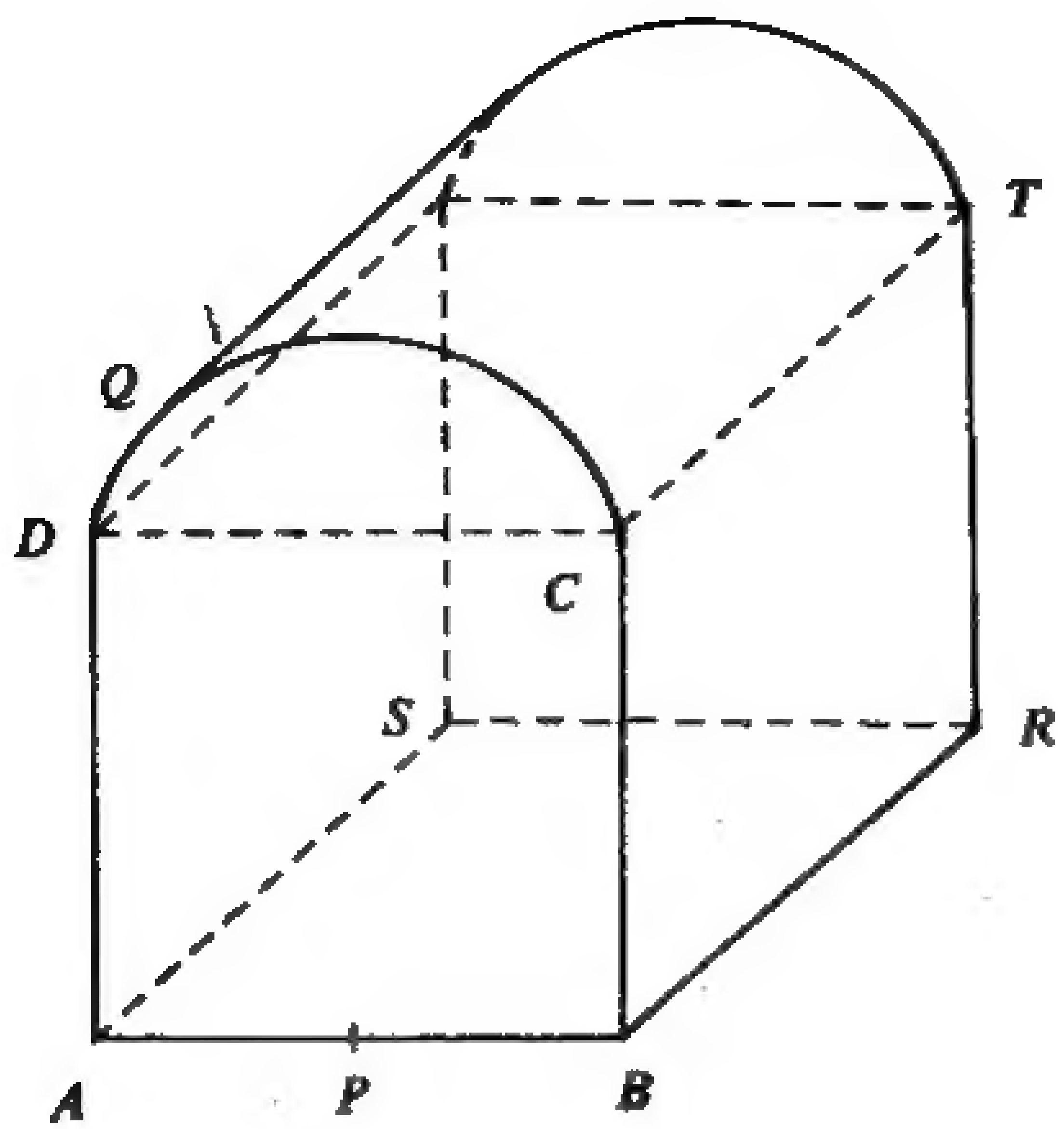

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(3 marks)

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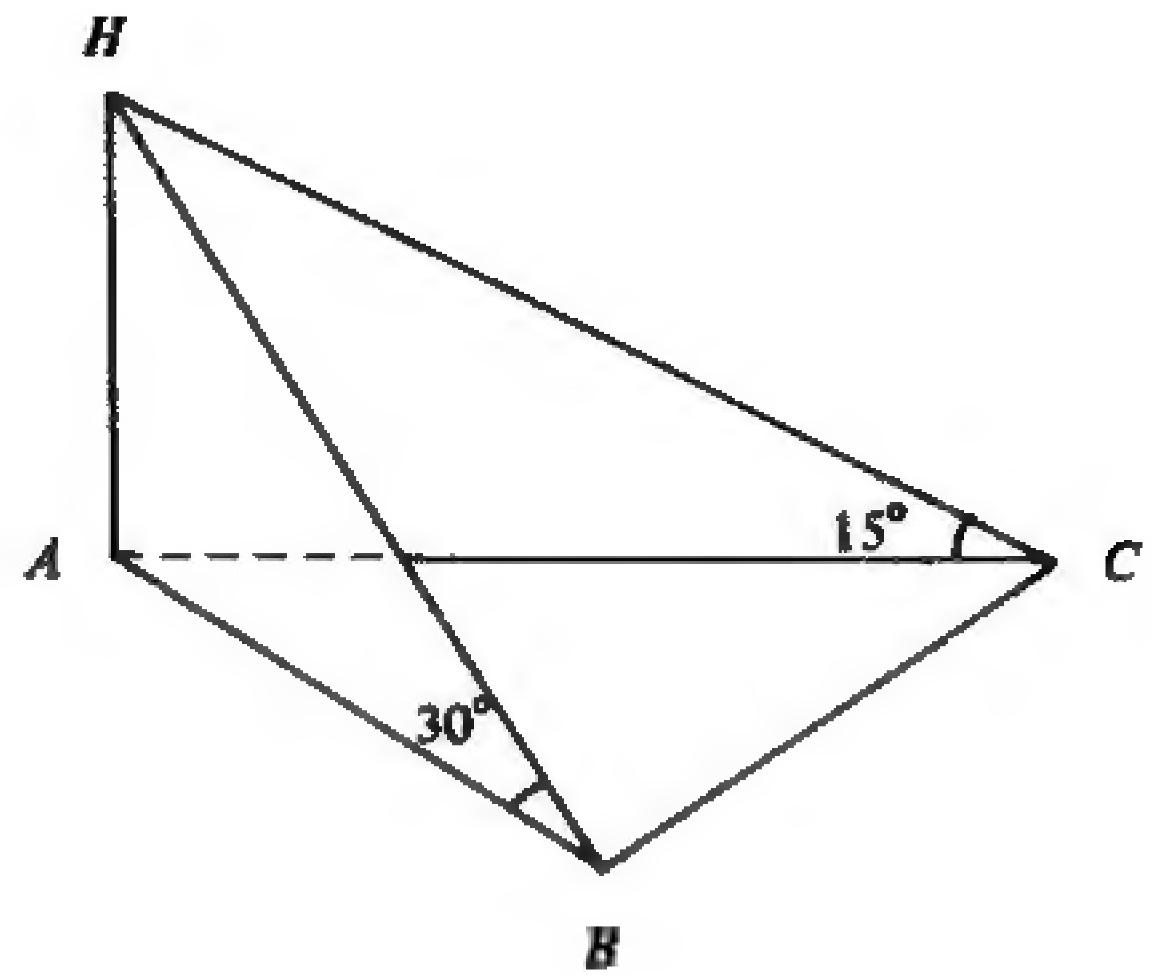
(3 marks)

13.



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(i) find, in m, the distance between B and C;

(ii) find, in m^2, the area of ΔABC.

(6 marks)

Go on to the next page

14. A school and a youth centre agree to share the total expenditure for a camp in the ratio $3 : 1$. The total expenditure $\$E$ for the camp is the sum of two parts: one part is a constant $\$C$, and the other part varies directly as the number of participants N . If there are 300 participants, the school has to pay \$7500. If there are 500 participants, the school has to pay \$12 000.

(a) Find the total expenditure for the camp, when the school has to pay \$7500.

(2 marks)

(b) Find the value of C .

(5 marks)

(c) Express E in terms of N .

(2 marks)

(d) If the youth centre has to pay \$4750, find the number of participants.

(3 marks)

END OF PAPER